The use of triangular (in addition to rectangular) subsections results in a more precise modeling of conductors of complex shapes, particularly when curved edges are involved. This in turn can lead to a reduction in the total number of subsections required for the analysis of a specific geometry, accompanied by a similar reduction in the order of the matrix requiring inversion.

As a consequence, a computational limitation on Patel's method may be alleviated to some degree. When consideration is given to the fact that the matrix will possibly be ill conditioned [2], a smaller number of subsections is doubly beneficial. It is also shown in [1] that for a given specified accuracy, a further reduction in the number of subsections can be achieved by irregular subdivision of conductors in a manner related to the expected charge distribution. Thus the smallest subsections are employed where the charge density is changing most rapidly with position, at conductor edges, for example.

References

[1] A. B. Birtles, B. J. Mayo, and A. W. Bennett, "A computer technique for solving three dimensional electron optics and capacitance problems," *Proc. Inst. Elec. Eng.*, vol. 120, no. 2, p. 213, Feb. 1973.
[2] C. W. Bostain and P. H. Wiley, "Concerning the moment solution for the charge distribution on a square conducting plate," *Proc. IEEE* (Lett.), vol. 59, p. 1639, Nov. 1971.

Comments on "An Analytical Equivalent Circuit Representation for Waveguide-Mounted Gunn Oscillators"

D. N. SINGH

In the above paper,[1] the authors have tried to explain the experimentally observed mode-switching phenomenon in waveguide-mounted Gunn oscillators on the basis of nonlinear behavior of the device. It has been shown in Fig. 6(a)[1] that the device susceptance decreases to zero at 8.18 GHz, as a result of which the condition of oscillation given by (4) cannot be satisfied. This leads to the mode switching observed at this frequency. In our opinion, the device susceptance cannot become zero over the entire frequency band of interest. It is not possible for the dynamic nonlinearities to make the device susceptance go to zero, even at the mode-switching frequency. In fact, the device susceptance will always remain capacitive, while the load susceptance presented to the device chip can become zero, inductive, or capacitive, depending upon the operating frequency and the particular circuit parameters. For steady-state oscillations, the load susceptance presented to the device chip should always be inductive, and the frequency switching may occur once the load susceptance becomes capacitive. However, the load conductance presented to the device chip should also be lower than the device conductance for the steady-state oscillations to build up. It is just possible that the load conductance may be more favorable for λg mode of operation rather than for $\lambda g/2$ mode of operation. This may also cause mode switching, as pointed out by Eisenhart and Khan [1]. It is highly probable that the mode switching is caused by this effect, rather than due to the nonlinearity of the device parameter as pointed out by Jethwa and Gunshor.

It has also been mentioned[1] that the mode switching also occurs in the case of reduced height waveguide cavities. The mode-switching frequency has been reported to be 9.3 GHz—a value much higher than in the case of full height waveguide. However, no mention has been made of the height of the waveguide used during these investigations. This is an important parameter, as the actual mode-switching frequency will be very much dependent on the height of the waveguide.

Thus it is clear from the above discussions that it is the characteristics of the circuit, and not of the device, which are responsible for the phenomenon of mode switching and it should be possible to avoid the mode switching over a frequency band of interest by suitable circuit design.

References

[1] R. L. Eisenhart and P. J. Khan, "Some tuning characteristics and oscillation conditions of a waveguide-mounted transferred-electron diode oscillator," *IEEE Trans. Electron Devices*, vol. ED-19, pp. 1050–1055, Sept. 1972.

Manuscript received February 8, 1973.
The author is with the Microwave Engineering Group, Tata Institute of Fundamental Research, Bombay, India.
[1] C. P. Jethwa and R. L. Gunshor, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 565–572, Sept. 1972.

Reply[2] by C. P. Jethwa[3] and R. L. Gunshor[4]

It is clearly stated in our paper that we have not tried to explain the experimentally observed mode switching on the basis of any single aspect of the device–circuit interaction. What we have done is to describe what the linear model indicates is occurring in the mode-switching regions of the tuning curves. The question of nonlinearity enters in the observation that in the region of mode switching, the experimentally observed tuning curves tend to pull up slightly from the curves predicted using the linear device mode. This effect, combined with the observation of nonsinusoidal device waveforms in this region, tends to support the suggestion that nonlinear aspects of the device behavior affect mode switching.

Singh makes the rather obvious comment that mode switching may take place from the $\lambda g/2$ to the λg mode as a switch to a more favorable load conductance. A careful reading of our paper will show that we also make this observation (see Section IV), and in fact we show that the load resistance rises rapidly just before mode switching (see our Fig. 7).

It is not difficult for us to think of the device as essentially capacitive from a physical point of view; however, the concept of steady-state susceptance is another matter. It is well known that the concept of device susceptance for a nonlinear device in which there may be other than the fundamental frequency present is not trivial. In fact, it can be shown that the device susceptance (at the fundamental frequency) is a function of the amplitude and phase of the other components. It is therefore not surprising to see the device susceptance vary with frequency in such a way as to suggest a large variation in "capacitance." We find that when the capacitive susceptance tends *toward* zero rapidly, this corresponding to the pulling up of the tuning curves (nonlinearities?), mode jumping occurs. In the paper by Tsai *et al.* [1], one can see a 2 to 1 variation in device susceptance corresponding to a 10-percent variation in frequency.

Finally, the reduced height waveguide is one half the full waveguide height.

References

[1] W. C. Tsai, F. J. Rosenbaum, and L. A. MacKenzie, "Circuit analysis of waveguide-cavity Gunn-effect oscillator," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 808–817, Nov. 1970.

[2] Manuscript received April 9, 1973.
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Trigonometric Functions and the Smith Chart

J. T. SENISE

***Abstract*—Sines and cosines can be read directly on the Smith chart.**

Smith, in his excellent book on the Smith chart [1], states that "the sine and cosine functions of α are not directly obtainable from the Smith chart."

Actually, sin α and cos α may be obtained very easily, as shown in Fig. 1.

A straight line joining $A(0, 0)$ to α ("angle of reflection coefficient") on the peripheral scale of the chart crosses the straight line through $B(1, 0)$ and $D(0, 1)$ at the point $P(p, q)$ of coordinates $p = \cos \alpha$, $q = \sin \alpha$.

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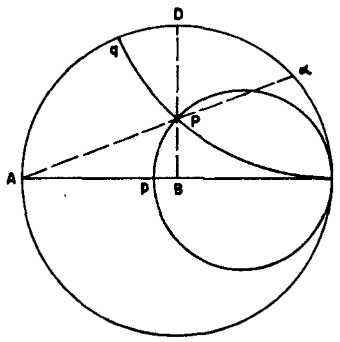

Fig. 1. Smith chart.

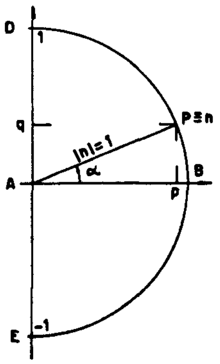

Fig. 2. n plane.

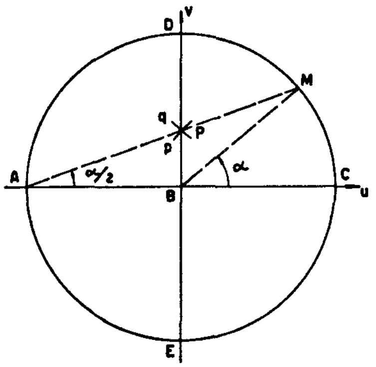

Fig. 3. w plane.

The proof is simple. The Smith chart originates from the conformal mapping of the plane of the normalized complex variable $n=p+jq$ over the plane of the complex variable $w=u+jv$, through the bilinear transformation $w=(n-1)/(n+1)$.

On plane n, the locus of points with unit modulus $|n|=1$, is the circle BDE (Fig. 2).

A point P on this circle can be represented either by its polar or rectangular coordinates as

$$P = n = 1 \cdot e^{j\alpha} = \cos\alpha + j\sin\alpha.$$

The image of the DBE circle on plane w is the segment DBE (Fig. 3).

Coordinates of P on the plane w are obtained directly from the transformation

$$w = u + jv = \frac{n-1}{n+1} = \frac{e^{j\alpha}-1}{e^{j\alpha}+1} = 0 + j\tan\frac{\alpha}{2}\cdot$$

Therefore, $\overline{PB}=\tan\alpha/2$, and since $\overline{AB}=1$, it follows that angle $(PAB)=\alpha/2$, and consequently angle $(MBC)=\alpha$.

This proves that segments AM and BD cross at point P corresponding to $p=\cos\alpha$, $q=\sin\alpha$.

References

[1] P. H. Smith, *Electronic Applications of the Smith Chart.* New York: McGraw-Hill, 1969, p. 166.

Comments on "A Quick Accurate Method to Measure the Dielectric Constant of Microwave Integrated-Circuit Substrates"

P. H. LADBROOKE, M. H. N. POTOK, AND E. H. ENGLAND

If, in the above short paper,[1] the data for open-edged resonators are plotted to a base of $\sqrt{m^2+n^2}$, where (n, m) characterizes the cavity mode, an excellent comparison with [1, Fig. 4(b)] is found. Such a plot is given in Fig. 1. It can therefore be asked whether Howell's frequency errors are of reactive origin, not resistive as he suggests. His results do not properly support the notion of resonant frequency change due to radiation loss, since there is no apparent correlation between the mode Q's and the frequency (or dielectric constant) errors for open sidewalls in Table I.

His results for closed-edge substrates are also consistent with reactive perturbation of the cavity. By positioning the apertures at the very corners, Howell seems to have achieved coupling to $\boldsymbol{H}$, without disturbing the electric field, in a region where the relative strength of $\boldsymbol{H}$ is mode independent, being essentially the boundary value. Hence, there should be an error due to magnetic perturbation of the same magnitude for all modes. Such a mode independence is shown by Howell's Table I[1]; a reasonable estimate for the associated error in ϵ would be $+1.5$ percent for closed sidewalls. The exact magnitude of this error depends upon the ratio of substrate thickness to coax conductor spacing, dielectric constant (i.e., how tightly the fields are bound to the dielectric), the coupling-aperture dimensions, and the detector sensitivity.

We have briefly examined a quartz slice of dimensions $75\times75\times2.5$ mm, prepared in the manner proposed by Howell. The results in Table I were obtained for the lowest few modes, to be compared with $\epsilon=3.85$ found by the methods in [1]. In this case, therefore, with approximately 2.5 mm bared at two diagonally opposite corners, the dominant effect was one of increased electric energy storage due to $\boldsymbol{E}$ no longer being zero at these corners. In view of the fact that in [1], for open sidewalls at least, the error was virtually identical for samples of Al_2O_3 and SiO_2 (being approximately five times thicker), it would be of interest to know the dimensions of Howell's corner apertures in his Al_2O_3 cavities such that his electric field was unaffected, whereas in our quartz this was no longer true.

Finally, if the central idea of our short paper is correct (namely that for this type of resonator, the frequency errors are dominated by reactive as opposed to resistive effects), then it should be possible to choose an optimum coupling point. Provided that perturbation of the electric field can be avoided, the best excitation for closed sidewalls probably is exactly that used by Howell; what recommends it is the error invariance among modes, so that only one or two spot measurements need be made. We have quickly tried this method with Al_2O_3 and N-type connectors and find the signal more difficult to detect than in the other techniques we have examined, due to exterior transmission paths as noted by Howell.

References

[1] P. H. Ladbrooke, M. H. N. Potok, and E. H. England, "Coupling errors in cavity resonance measurements on MIC dielectrics," this issue, pp. 560–562.

Manuscript received February 23, 1973.
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[1] J. Q. Howell, *IEEE Trans. Microwave Theory Tech.* (Short Paper), vol. MTT-21, pp. 142–143, Mar. 1973.